ABI 2011

Analysis
Aufgabengruppe I

Teil 1

1) $f(x) = \frac{2x+3}{4x+5}$; $\mathbb{D} = \mathbb{R} \setminus \{-\frac{5}{4}\}$

$f'(x) = \frac{(4x+5)\cdot 2 - (2x+3)\cdot 4}{(4x+5)^2} = \frac{8x+10-8x-12}{(4x+5)^2} = -\frac{2}{(4x+5)^2}$

2) $F(x) = \frac{1}{4}x^2 \cdot (2\ln x - 1)$ $\mathbb{D} = \mathbb{R}^+$

$f(x) = x \cdot \ln x$ $\mathbb{D} = \mathbb{R}^+$

F ist Stammfunktion von f wenn gilt: $F'(x) = f(x)$

$F'(x) = \frac{1}{4} \cdot 2x \cdot (2\ln x - 1) + \frac{1}{4}x^2 \cdot 2 \cdot \frac{1}{x}$

$= \frac{1}{2}x \cdot 2\ln x - \frac{1}{2}x + \frac{1}{2}x = x\ln x = f(x)$ ged

Menge aller Stammfunktionen:

$F(x) = \frac{1}{4}x^2(2\ln x - 1) + c$

$F(1) = 0$ (Nullstelle bei $x = 1$)

$\frac{1}{4}\cdot 1^2(2\underbrace{\ln 1}_{=0} - 1) + c = 0$

$-\frac{1}{4} + c = 0 \Rightarrow c = \frac{1}{4}$

$\Rightarrow F(x) = \frac{1}{4}x^2(2\ln x - 1) + \frac{1}{4}$

3) $N(x) = N_0 \cdot e^{k\cdot(x-2000)}$ (in Milliarden)

$N_0 = 6{,}1$ (Mrd)

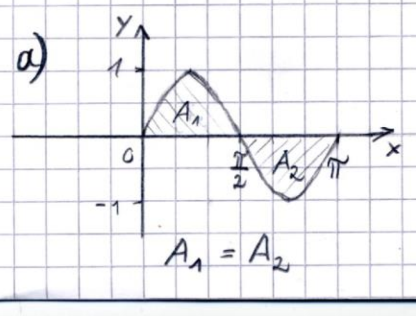
$N(2010) = 6{,}1 \cdot e^{k(2010-2000)} = 6{,}9$

$e^{10k} = \frac{6{,}9}{6{,}1}$ | ln

$10k = \ln\frac{6{,}9}{6{,}1} \Rightarrow k = \frac{1}{10}\cdot\ln\frac{6{,}9}{6{,}1} \approx 0{,}012$

4) $\int_0^{\pi} \sin(2x)dx = 0$

a)



$A_1 = A_2$

b) $\int_0^{\pi} \sin(2x)dx$

$= \left[-\frac{1}{2}\cos(2x)\right]_0^{\pi} \left(= -\frac{1}{2}\left[\cos(2x)\right]_0^{\pi}\right)$

$= -\frac{1}{2}\cos(2\pi) - (-\frac{1}{2}\cos 0)$

$= -\frac{1}{2}\cdot 1 - (-\frac{1}{2}\cdot 1) = -\frac{1}{2} + \frac{1}{2} = 0$

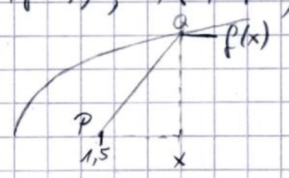
Teil 2

1) $f(x) = \sqrt{x+3}$; D_f

a) $x+3 \overset{!}{\geq} 0 \iff x \geq -3 \implies D_f = [-3; +\infty[$

G_f geht aus G_w durch Verschiebung von 3 LE nach links entlang der x-Achse hervor.

b) $d(x) = \overline{PQ}$ $Q(x/f(x))$, $P(1{,}5/0)$



Pythagoras:

$(d(x))^2 = (x-1{,}5)^2 + (f(x)-0)^2$

$(d(x))^2 = x^2 - 3x + 2{,}25 + x + 3 = x^2 - 2x + 5{,}25$

$\implies d(x) = \sqrt{x^2-2x+5{,}25}$

c) $d(x)$ soll minimal sein (TIP)

$d'(x) = \frac{2x-2}{2\sqrt{x^2-2x+5{,}25}} = \frac{2(x-1)}{2\sqrt{x^2-2x+5{,}25}} = \frac{x-1}{\sqrt{x^2-2x+5{,}25}}$

$d'(x) \overset{!}{=} 0 \iff x-1=0 \implies x=1$

für $x<1$ gilt: $d'(x) = \frac{\overbrace{x-1}^{<0}}{\underbrace{\sqrt{x^2-2x+5{,}25}}_{>0}} < 0 \implies d$ abnehmend

für $x>1$ gilt: $d'(x) = \frac{\overbrace{x-1}^{>0}}{\underbrace{\sqrt{x^2-2x+5{,}25}}_{>0}} > 0 \implies d$ zunehmend

$d(1) = \sqrt{1-2+5{,}25} \approx 2{,}06$

Randwerte: $x = -3$ $d(-3) = 4{,}5$

$\implies$ bei $x=1$ hat $d(x)$ ein Minimum

$f(1) = \sqrt{1+3} = 2 \implies Q_E(1/2)$

d) Tangente an G_f in $Q_E(1/2)$: $y = f'(x_0)(x-x_0)+f(x_0)$

$f'(x) = \frac{1}{2\sqrt{x+3}}$; $m_t = f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\left[y = \frac{1}{2\sqrt{1+3}} \cdot (x-1) + 2 = \frac{1}{4}x - \frac{1}{4} + 2 = \frac{1}{4}x + 1{,}75 \implies m_t = \frac{1}{4} \right]$

Gerade PQ: $m_{PQ} = \frac{2-0}{1-1{,}5} = \frac{2}{-0{,}5} = -4$

$m_t \cdot m_{PQ} = \frac{1}{4} \cdot (-4) = -1 \implies t \perp PQ$

e) $A = \int_{-3}^{1} \sqrt{x+3}\, dx + \frac{1}{2} \cdot \frac{1}{2} \cdot 2$ $\qquad (x+3)^{\frac{1}{2}}$

$= \left[\frac{2}{3} \cdot (x+3)^{\frac{3}{2}} \right]_{-3}^{1} + \frac{1}{2}$

$= \frac{2}{3} \cdot (1+3)^{\frac{3}{2}} - \frac{2}{3}(-3+3)^{\frac{3}{2}} + \frac{1}{2} = \frac{2}{3} \cdot 8 + \frac{1}{2} = \frac{35}{6}$ FE

2) a) $g'(-1) \approx 2 = \frac{\Delta y}{\Delta x}$

$\lim_{x \to \pm\infty} (g'(x)) = \frac{1}{2}$

b) I) Die schräge Asymptote des Graphen von I hat die Gleichung $y = x - 1$ und ist nicht identisch mit der von g

II) Die Polstelle von II ist mit Vorzeichenwechsel

$g(x) = \frac{1}{2}x - 1 + \frac{a}{(x-1)^2}$

$g(-1) \overset{!}{=} 0$

$\frac{1}{2} \cdot (-1) - 1 + \frac{a}{(-1-1)^2} = 0$

$-1,5 + \frac{a}{4} = 0$

$\frac{a}{4} = 1,5 \Rightarrow a = 6$

c) $h(x) = \ln(g(x))$

$g(x) \overset{!}{>} 0 \Rightarrow \mathbb{D}_h =]-1; 1[\cup]1, +\infty[$

$\lim_{x \to -1^+} h(x) = -\infty$

$\lim_{x \to 1^-} h(x) = +\infty$, $\lim_{x \to 1^+} h(x) = +\infty$

$\lim_{x \to +\infty} h(x) = +\infty$

Nullstelle von h $:\hat{=}$ $g(x) = 1$

$x_{h_0} \approx -0,6$

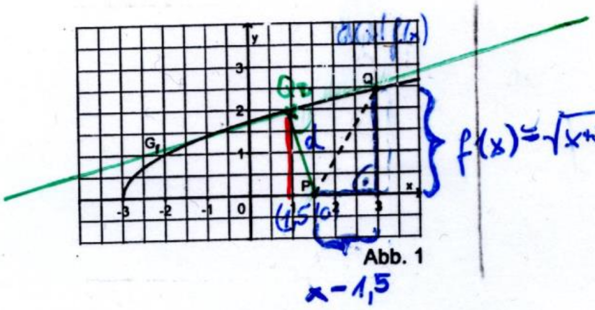


Abb. 1

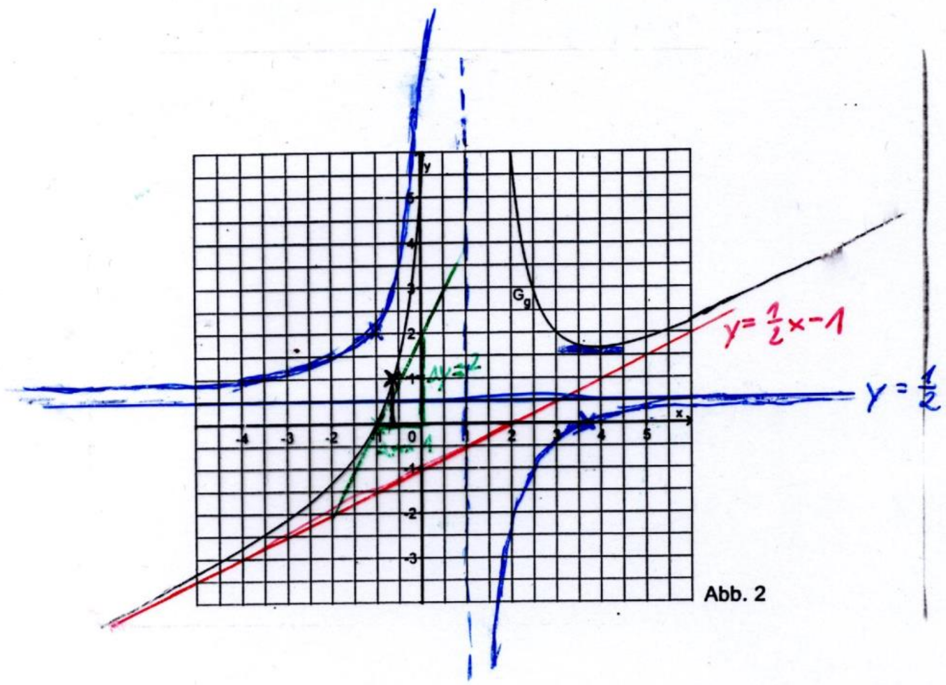


Abb. 2