ABI 2015
Geometrie
Aufgabengruppe 1

Teil A

1) $A(0|1|2)$; $B(2|5|6)$

a) $|\overrightarrow{AB}| = \left|\begin{pmatrix}2-0\\5-1\\6-2\end{pmatrix}\right| = \left|\begin{pmatrix}2\\4\\4\end{pmatrix}\right| = \sqrt{2^2+4^2+4^2} = \sqrt{36} = 6 = d(A;B)_{qed}$

$\vec{C} = \vec{A} + 2\cdot\overrightarrow{AB} = \begin{pmatrix}0\\1\\2\end{pmatrix} + 2\cdot\begin{pmatrix}2\\4\\4\end{pmatrix} = \begin{pmatrix}4\\9\\10\end{pmatrix} \Rightarrow C(4|9|10)$

$\vec{D} = \vec{A} - 2\cdot\overrightarrow{AB} = \begin{pmatrix}0\\1\\2\end{pmatrix} - 2\cdot\begin{pmatrix}2\\4\\4\end{pmatrix} = \begin{pmatrix}-4\\-7\\-6\end{pmatrix} \Rightarrow D(-4|-7|-6)$

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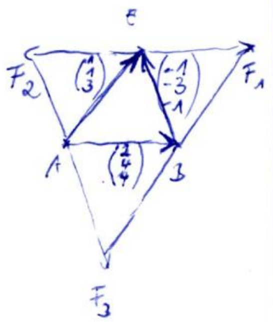

b) $\vec{F_1} = \vec{E} + \overrightarrow{AB} = \begin{pmatrix}1\\2\\5\end{pmatrix} + \begin{pmatrix}2\\4\\4\end{pmatrix} = \begin{pmatrix}3\\6\\9\end{pmatrix} \Rightarrow F_1(3|6|9)$

$\vec{F_2} = \vec{E} - \overrightarrow{AB} = \begin{pmatrix}1\\2\\5\end{pmatrix} - \begin{pmatrix}2\\4\\4\end{pmatrix} = \begin{pmatrix}-1\\-2\\1\end{pmatrix} \Rightarrow F_2(-1|-2|1)$

oder:

$\vec{F_3} = \vec{A} + \overrightarrow{EB} = \begin{pmatrix}0\\1\\2\end{pmatrix} + \begin{pmatrix}2-1\\5-2\\6-5\end{pmatrix} = \begin{pmatrix}1\\4\\3\end{pmatrix} \Rightarrow F_3(1|4|3)$

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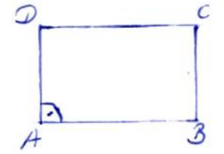
2) $A(0|0|0)$; $B(4|4|2)$; $C(8|0|2)$; $D(4|-4|0)$; $S(1|1|-4)$



a) $\overrightarrow{AB} = \begin{pmatrix}4-0\\4-0\\4-0\end{pmatrix} = \begin{pmatrix}4\\4\\2\end{pmatrix}$; $\overrightarrow{AD} = \begin{pmatrix}4-0\\-4-0\\0-0\end{pmatrix} = \begin{pmatrix}4\\-4\\0\end{pmatrix}$

$\overrightarrow{AB} \circ \overrightarrow{AD} = \begin{pmatrix}4\\4\\2\end{pmatrix} \circ \begin{pmatrix}4\\-4\\0\end{pmatrix} = 16 + (-16) + 0 = 0 \Rightarrow \overrightarrow{AB} \perp \overrightarrow{AD}$

$\Rightarrow$ ABCD ist ein Parallelogramm

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b) $A_{ABCD} = 24\sqrt{2} = G$

$|\overrightarrow{AS}| = \left|\begin{pmatrix}1-0\\1-0\\-4-0\end{pmatrix}\right| = \sqrt{1^2+1^2+(-4)^2} = \sqrt{18} = 3\sqrt{2} = h$

$V_P = \frac{1}{3}\cdot G\cdot h = \frac{1}{3}\cdot 24\sqrt{2}\cdot 3\sqrt{2} = 48\ VE$

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Teil B

$E: x_1 + x_3 = 2$; $A(0|\sqrt{2}|2)$; $g: \vec{x} = \begin{pmatrix} 0 \\ \sqrt{2} \\ 2 \end{pmatrix} + \lambda \cdot \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}$ $\lambda \in \mathbb{R}$

a) E liegt parallel zur x_2-Achse ✓

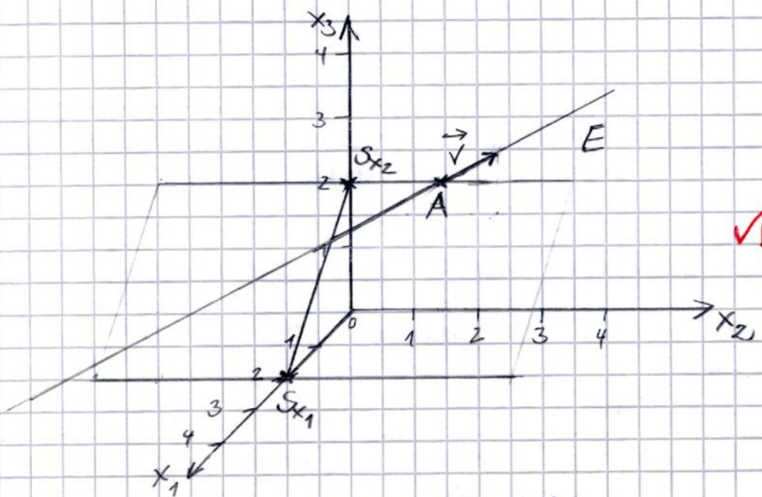
g in E: $(0-\lambda) + (2+\lambda) = -\lambda + 2 + \lambda = 2$ ✓ $\Rightarrow g \in E$ ✓

SP mit der x_1-Achse: $x_2 = 0$; $x_3 = 0$

$x_1 + 0 = 2 \Rightarrow S_{x_1}(2|0|0)$ ✓ nur $x_1 = 2$ $-0{,}5$

SP mit der x_3-Achse: $x_1 = 0$; $x_2 = 0$

$0 + x_3 = 2 \Rightarrow S_{x_3}(0|0|2)$ ✓



✓✓

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b) $\vec{n}_{E_{x_1x_2}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$; $\sin\alpha = \dfrac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \circ \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix}}{\sqrt{0^2+0^2+1^2} \cdot \sqrt{(-1)^2 + \sqrt{2}^2 + 1^2}} = \dfrac{1}{1 \cdot 2} = \dfrac{1}{2}$ ✓ ✓

$\Rightarrow \alpha = \arcsin\left(\frac{1}{2}\right) = 30°$ ✓

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c) $M(0|3\sqrt{2}|2)$

$\vec{MB} \circ \vec{v} \overset{!}{=} 0$ $B \in g$

$\begin{pmatrix} 0-\lambda-0 \\ \sqrt{2}+\sqrt{2}\lambda - 3\sqrt{2} \\ 2+\lambda-2 \end{pmatrix} \circ \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} = 0$ ✓ $\Leftrightarrow \begin{pmatrix} -\lambda \\ -2\sqrt{2} + \sqrt{2}\lambda \\ \lambda \end{pmatrix} \circ \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} = 0$

$\Leftrightarrow \lambda - 4 + 2\lambda + \lambda = 0$ ✓

$4\lambda = 4 \Rightarrow \lambda = 1$ ✓

(3)

oder: Hilfsebene H mit $g \perp H$ und $M \in H$:

$$H: \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} \circ \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} 0 \\ 3\sqrt{2} \\ 2 \end{pmatrix} \right] = 0 \quad ✓$$

$$\Leftrightarrow \quad -x_1 + \sqrt{2}x_2 + x_3 - (6+2) = 0$$

$$H: -x_1 + \sqrt{2}x_2 + x_3 - 8 = 0 \quad ✓$$

$$g \cap H: \; -(0-\lambda) + \sqrt{2} \cdot (\sqrt{2} + \sqrt{2}\lambda) + (2+\lambda) - 8 = 0$$

$$\lambda + 2 + 2\lambda + 2 + \lambda - 8 = 0$$

oder (3) $$4\lambda - 4 = 0 \Rightarrow \lambda = 1 \quad ✓$$

$$\overset{\lambda \text{ in } g}{\Longrightarrow} \vec{B} = \begin{pmatrix} 0 \\ \sqrt{2} \\ 2 \end{pmatrix} + 1 \cdot \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2\sqrt{2} \\ 3 \end{pmatrix} \Rightarrow B(-1 \mid 2\sqrt{2} \mid 3) \quad ✓$$

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$$r = |\overrightarrow{MB}| = \left| \begin{pmatrix} -1 - 0 \\ 2\sqrt{2} - 3\sqrt{2} \\ 3 - 2 \end{pmatrix} \right| = \left| \begin{pmatrix} -1 \\ -\sqrt{2} \\ 1 \end{pmatrix} \right| = \sqrt{(-1)^2 + (-\sqrt{2})^2 + 1^2} = 2 \quad ✓$$

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d) Da $\overrightarrow{MB}$ senkrecht zu g und damit zu $\vec{v}$ ist, bei einem Viertelkreis zudem $\overrightarrow{MC} \perp \overrightarrow{MB}$, ist $\overrightarrow{MC} \parallel \vec{v}$. ✓

Da $|\vec{v}| = \sqrt{(-1)^2 + \sqrt{2}^2 + 1^2} = \sqrt{4} = 2 = r$ gilt $\overrightarrow{MC} = \vec{v}$ ✓

2 und damit $\vec{C} = \vec{M} + \overrightarrow{MC} = \vec{M} + \vec{v}$

e) $v = 15\frac{m}{s} \qquad 1LE \mathrel{\hat{=}} 10m$

$$\overline{AB} = |\overrightarrow{AB}| = \left| \begin{pmatrix} -1 - 0 \\ 2\sqrt{2} - \sqrt{2} \\ 3 - 2 \end{pmatrix} \right| = \left| \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} \right| = \sqrt{1+2+1} = 2 \quad ✓$$

$$\frac{1}{4} \cdot 2\pi r = \frac{1}{4} \cdot 2\pi \cdot 2 = \pi \quad ✓$$

Länge der Gesamtstrecke: $s = (2+\pi) \cdot 10m$

4 $$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{(2+\pi) \cdot 10m}{15\frac{m}{s}} ✓ \approx 3{,}4s \quad ✓$$

−0,5 bei falscher Rundung