S.36/3

Berechnen Sie den Inhalt der Fläche zwischen dem Graphen von f und der x-Achse über dem angegebenen Intervall.

a) $f(x)=\frac{1}{2}x^3-2x; \qquad I=[-1;3]$

Nullstellen von f:

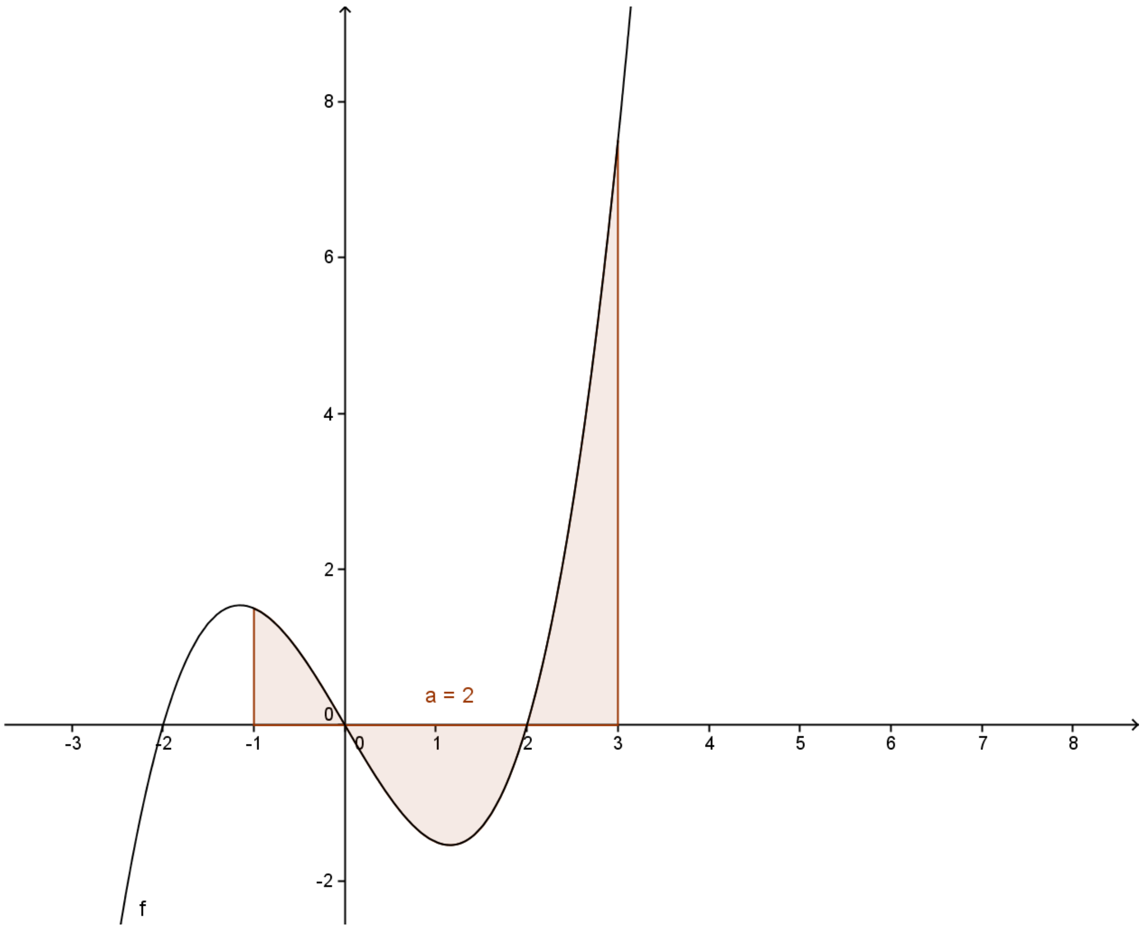
$$f(x)=0 \Leftrightarrow \frac{1}{2}x(x^2-4)=0 \Leftrightarrow \frac{1}{2}x(x-2)(x+2)=0 \qquad \Rightarrow x_1=0; x_2=2; x_3=-2 \notin I$$

Fläche:

$$A=\left|\int_{-1}^{0} f(x)dx\right|+\left|\int_{0}^{2} f(x)dx\right|+\left|\int_{2}^{3} f(x)dx\right|=$$

$$\left|\left[\frac{1}{8}x^4-x^2\right]_{-1}^{0}\right|+\left|\left[\frac{1}{8}x^4-x^2\right]_{0}^{2}\right|+\left|\left[\frac{1}{8}x^4-x^2\right]_{2}^{3}\right|=$$

$$\left|0-\left(\frac{1}{8}\cdot(-1)^4-(-1)^2\right)\right|+\left|\frac{1}{8}\cdot 2^4-2^2-0\right|+\left|\frac{1}{8}\cdot 3^4-3^2-\left(\frac{1}{8}\cdot 2^4-2^2\right)\right|=0{,}875+|-2|+3{,}125=6$$



b) $f(x) = -2x^2 - 2x + 4;\ I = [-3;3]$

Nullstellen von f:

$$f(x) = 0 \Leftrightarrow -2x^2 - 2x + 4 = 0 \Rightarrow x_{1/2} = \frac{2 \pm \sqrt{4 - 4 \cdot (-2) \cdot 4}}{-4} = \frac{2 \pm \sqrt{36}}{-4} \qquad \Rightarrow x_1 = -2; x_2 = 1;$$

Fläche:

$$A = \left|\int_{-3}^{-2} f(x)dx\right| + \left|\int_{-2}^{1} f(x)dx\right| + \left|\int_{1}^{3} f(x)dx\right| =$$

$$\left|\left[-\frac{2}{3}x^3 - x^2 + 4x\right]_{-3}^{-2}\right| + \left|\left[-\frac{2}{3}x^3 - x^2 + 4x\right]_{-2}^{1}\right| + \left|\left[-\frac{2}{3}x^3 - x^2 + 4x\right]_{1}^{3}\right| =$$

$$\left|-\frac{2}{3}\cdot(-8) - 4 - 8 - \left(-\frac{2}{3}\cdot(-27) - 9 - 12\right)\right| + \left|-\frac{2}{3} - 1 + 4 - \left(-\frac{2}{3}\cdot(-8) - 4 - 8\right)\right| + \left|-\frac{2}{3}\cdot 27 - 9 - 12 - \left(-\frac{2}{3} - 1 + 4\right)\right| =$$

$$\left|-\frac{20}{3} - (-3)\right| + \left|\frac{7}{3} - \left(-\frac{20}{3}\right)\right| + \left|-15 - \frac{7}{3}\right| = \left|-3\frac{2}{3}\right| + 9 + \left|-17\frac{1}{3}\right| = 30$$