S. 168/8

a) $(x^2+x)\cdot e^x \overset{!}{=} 0 \iff x\cdot(x+1)\cdot \underbrace{e^x}_{\neq 0} = 0$

$x_{01} = 0;\ x_{02} = -1$

abde

b) $e^{3x} - e^x = 0 \iff e^x(e^{2x}-1) = 0$

$z^3 - z = 0$

$\underbrace{e^x}_{\neq 0} \cdot (e^x - 1)\cdot \underbrace{(\underbrace{e^x}_{>0} + 1)}_{>1} = 0$

$\neq 1$

$e^x = 1 \Rightarrow x_0 = 0$

c) $\dfrac{x^3 e^{2x} - x^2 e^{2x}}{\underbrace{3x + 2e^{2x}}_{\neq 0}} = 0 \iff x^2 \underbrace{e^{2x}}_{>0} (x-1) = 0$

$x_{01} = 0;\ x_{02} = 1$

d) $\dfrac{\ln(x^2+2x-2)}{\underbrace{x^2+4}_{\neq 0}} = 0 \iff \ln(x^2+2x-2) = 0 \quad | e^{\nearrow}$ oder $\ln 1 = 0$

$x^2 + 2x - 2 = 1$

$x^2 + 2x - 3 = 0$

$x_{01/02} = \dfrac{-2 \pm \sqrt{4+12}}{2} \Rightarrow x_{01} = -3;\ x_{02} = 1$

e) $e^x - 2e^{-x} \overset{!}{=} 0 \quad | \cdot e^x$

$e^{2x} - 2 = 0$ subst: $e^x = z$ $z^2 - 2 = 0$

$(e^x)^2$

$(z-\sqrt{2})(z+\sqrt{2}) = 0$

$z_1 = \sqrt{2}$ oder $z_2 = -\sqrt{2}$

$(e^x - \sqrt{2})(e^x + \sqrt{2}) = 0$ Resubst. $e^x = \sqrt{2}$ $e^x = -\sqrt{2}$ ↯

$e^x = \sqrt{2} \vee e^x = -\sqrt{2}$

$x_{01} = \ln\sqrt{2}$ keine Lsg

$= \frac{1}{2}\ln 2$

↯

$x_{01} = \ln\sqrt{2}$

$= \frac{1}{2}\ln 2$ keine Lsg

f) $f(x) = x - 1 - \frac{1}{x} = 0 \quad | \cdot x$

$x^2 - x - 1 = 0$

$x_{01/02} = \dfrac{1 \pm \sqrt{1+4}}{2} = \quad x_{01} = \dfrac{1-\sqrt{5}}{2};\ x_{02} = \dfrac{1+\sqrt{5}}{2}$

S. 169/11

$f(x) = 2xe^{1-x} \qquad D_f = \mathbb{R}$

a) $f(x) \overset{!}{=} 0 \Leftrightarrow 2x\underbrace{e^{1-x}}_{\neq 0} = 0 \Rightarrow x_0 = 0$

b) $\lim\limits_{x \to -\infty} (\underbrace{2x}_{\to -\infty}\underbrace{e^{\overbrace{1-x}^{\to +\infty}}}_{\to +\infty}) = -\infty, \qquad \lim\limits_{x \to +\infty} (\underbrace{2x}_{\to +\infty}\underbrace{e^{\overbrace{1-x}^{\to -\infty}}}_{\to 0}) = 0$

c) $f'(x) = 2 \cdot e^{1-x} + 2xe^{1-x}(-1) = 2e^{1-x} \cdot (1-x)$

$f''(x) = 2e^{1-x}(-1) \cdot (1-x) + 2e^{1-x} \cdot (-1)$

$= -2e^{1-x}(1-x+1) = -2e^{1-x}(2-x)$

$f'(x) \overset{!}{=} 0 \Leftrightarrow \underbrace{2e^{1-x}}_{\neq 0}(1-x) = 0 \Rightarrow x = 1$

$f''(1) = -2\underbrace{e^{0}}_{1}(2-1) = -2 < 0 \Rightarrow$ Max. bei $x = 1$

$f(1) = 2 \cdot 1 \cdot e^{0} = 2 \Rightarrow HOP(1|2)$

d) $f''(x) = 0 \Leftrightarrow \underbrace{-2e^{1-x}}_{\neq 0}(2-x) = 0 \Rightarrow x = 2$

für $x < 2$ gilt: $f''(x) = \underbrace{-2}_{<0}\underbrace{e^{1-x}}_{>0}\underbrace{(2-x)}_{>0} < 0$

$\Rightarrow G_f$ ist rechtsgekrümmt in $]-\infty; 2[$

für $x > 2$ gilt: $f''(x) = \underbrace{-2}_{<0}\underbrace{e^{1-x}}_{>0}\underbrace{(2-x)}_{<0} > 0$

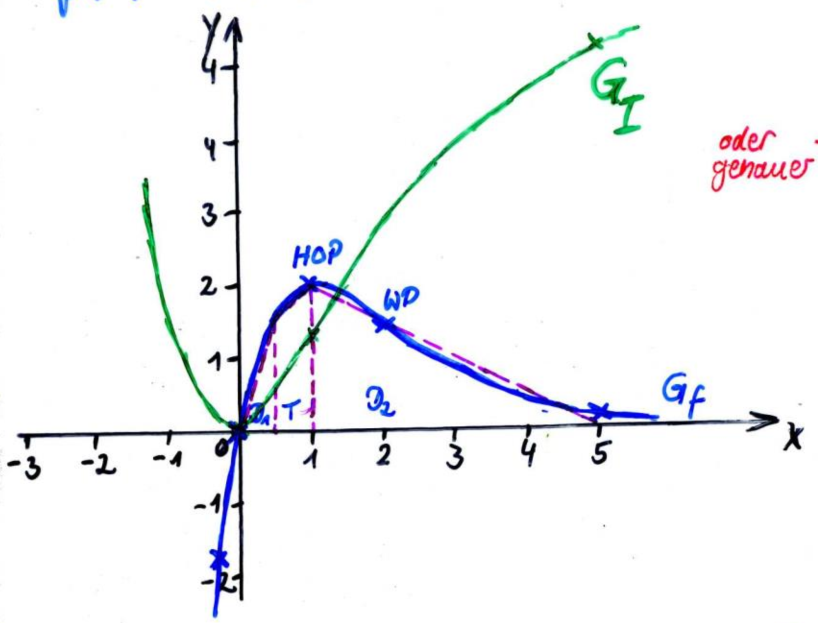
$\Rightarrow G_f$ ist linksgekrümmt in $]2; +\infty[$

$f(2) = 4e^{-1}$
$\Rightarrow WP(2|4e^{-1})$
$\approx (2|1{,}47)$

S. 169/11

e) $f(-0{,}25) = 2\cdot(-0{,}25)\cdot e^{1+0{,}25} = -0{,}5\cdot e^{1{,}25} \approx -1{,}75$

$f(5) = 2\cdot 5\cdot e^{1-5} = 10\cdot e^{-4} \approx 0{,}18$



f) $\int_0^1 f(x)\,dx \approx A_{D_1} + A_T$

oder genauer → $= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2} + \frac{1}{2}\cdot\left(\frac{3}{2}+\frac{4}{2}\right)\cdot\frac{1}{2}$ (1,65) (1,65)

$= \frac{3}{8} + \frac{7}{8} = \frac{10}{8} = 1{,}25$ (1,3)

$\int_1^5 f(x)\,dx \approx A_{D_2} = \frac{1}{2}\cdot 4\cdot 2 = 4$

g) $I(x) = \int_0^x f(t)\,dt$ mit $I'(x) = f(x) \Rightarrow$

→ I hat an der Stelle einen Extrempunkt, wo f eine Nullstelle mit VZW hat: $I'(0) = 0 = f(0)$ mit VZW von − nach +

$\Rightarrow$ Tiefpunkt $(0|0)$; $I(1) \approx 1{,}25$; $I(5) = 1{,}25 + 4 = 5{,}25$